

League Championship Algorithm (LCA): An algorithm for global optimization inspired by sport championships

Ali Husseinzadeh Kashan

Department of Industrial Engineering, Faculty of Engineering, Tarbiat Modares University, Tehran, Iran.

E-mail address: a.kashan@modares.ac.ir
a.kashani@aut.ac.ir

Abstract

League Championship Algorithm (LCA) is a recently proposed stochastic population based algorithm for continuous global optimization which tries to mimic a championship environment wherein artificial teams play in an artificial league for several weeks (iterations). Given the league schedule in each week, a number of individuals as sport teams play in pairs and their game outcome is determined in terms of win or loss (or tie), given known the playing strength (fitness value) along with the intended team formation/arrangement (solution) developed by each team. Modeling an artificial match analysis, each team devises the required changes in its formation (generation of a new solution) for the next week contest and the championship goes on for a number of seasons (stopping condition). An add-on module based on modeling the end season transfer of players is also developed to possibly speed up the global convergence of the algorithm. The aim of this paper is to describe LCA in a step by step manner together with verifying the rationale of the algorithm and suitability of the updating equations empirically.

Keywords: global optimization, numerical optimization, metaheuristic algorithms, league championships algorithm.

1. Introduction

Since 1970s that the idea of a general algorithmic framework emerged, many algorithms have been introduced. With the aim of being applicable with relatively few modifications to different optimization problems, these algorithms have got their source of inspiration from nature, society, culture, politics, human, etc. The term “metaheuristic” is used for such methods that combine rules and randomness while imitating natural, social, cultural and political phenomena. These methods are from now on regularly employed in all sectors of business, industry, engineering, etc. However, besides all of the interest necessary to applications of metaheuristics for solving difficult and complex optimization problems, occasionally a new metaheuristic algorithm is introduced which uses a novel metaphor as a guide for solving optimization problems. For example, particle swarm optimization algorithm (PSO) [1], introduced in 1995, models the flocking behavior of birds; harmony search (HS) [2], introduced in 2001, is conceptualized using the musical process of searching for a perfect state of harmony; bacterial foraging optimization algorithm (BFOA) [3], introduced in 2002, models foraging as an optimization process where an animal seeks to maximize energy per unit time spent for foraging; artificial bee colony algorithm (ABC) [4], introduced in 2005, simulates the intelligent foraging behavior of a honeybee swarm; central force optimization algorithm (CFO) [5], introduced in 2007, makes an analogy between the process of searching a decision space for the maxima of an objective function and flying probes through 3-dimensional physical space under the influence of gravity; fire fly algorithm (FA) [6], introduced in 2007, performs based on the idealization of the flashing characteristics of fireflies; group search optimizer (GSO) [7], introduced in 2009, simulates the animal searching behavior (an active movement by which animals find or attempt to find resources



such as food, mates, oviposition, or nesting sites); krill herd algorithm (KH) [8], introduced in 2012, works based on the simulation of the herding of the krill swarms in response to specific biological and environmental processes; and optics inspired optimization (OIO) [9], introduced in 2013, treats the surface of the numerical function to be optimized as a reflecting surface in which each peak is assumed to reflect as a convex mirror and each valley to reflect as a concave one.

The League Championship Algorithm (LCA) is a recently proposed algorithm for global optimization, which mimics the championship process in sport leagues [10]. Beside the nature, culture, politics, human, etc as the typical sources of inspiration of various algorithms, the metaphor of sporting competitions is used for the first time in LCA. The methodology of LCA can be described as follows. A number of individuals making role as sport teams compete in an artificial league for several weeks (iterations). Based on the league schedule in each week, teams play in pairs and their game outcome is determined in terms of win or loss (or tie), given the playing strength (fitness value) along with the particular team formation/arrangement (solution) followed by each team. Keeping track of the previous week events, each team devises the required changes in its formation/playing style (a new solution is generated) for the next week contest and the championship goes on for a number of seasons (stopping condition). LCA is a population based algorithm where its “teams” are similar to PSO’s “particles” but with a quite different way of performing their search. The way in which a new solution associated to an LCA’s team is generated is governed via imitating the match analysis process followed by coaches to design a suitable arrangement for their forthcoming match. In a typical match analysis, coaches will modify their arrangement on the basis of their own game experiences and their opponent’s style of play.

The above rationale is modeled by some LCA-specific paradigm as follows. To determine the winner/loser individuals to bias the search toward/outward them, LCA focuses on the relative comparison of individuals, and not their absolute fitness gains. Such a mechanism ensures that the win portion for the better solution (team) is greater than the win portion of the weaker solution (team). Therefore the search direction is expected to be toward winner (more likely the better solution) and in opposition of loser (more likely the weaker solution). Such a mechanism allows that the algorithm moves the population toward promising areas and at the same time escapes from local or fruitless areas. Modeling artificial match analysis mathematically in LCA yields four equations which generate new solutions in the search space. The development of these equations is closely related to develop a balance between intensification and diversification. Unlike many algorithms which only allow a given solution approaches to better solutions in the search space, LCA also allows retreat from solutions in a scheduled manner (supplied by the league schedule module). For this reason, we will see that LCA performs well on various types of problems. To preserve diversity and avoid premature convergence, LCA uses a truncated geometric probability distribution to choose the number of elements in a given solution that their value should change via one of the four equations to generate a new solution. Using a truncated geometric distribution we can set the number of changes dynamically with more emphasis given to the smaller or larger rate of changes.

This paper first introduces the rationale of LCA and then examines the role of updating equations of LCA in the sense that whether these equations have a lump sum significant effect or we can achieve the same or better performance with a subset of these equations. We then examine whether different learning strategies followed in the artificial post-match analysis of LCA are crucial or not. We finally try to speed up the convergence of basic LCA via 1) allowing the tie outcome happens beside the win or loss outcomes which are typically allowed in the basic LCA. Five scenarios are considered for interpretation of ties and updating equations are adapted accordingly 2) introducing the end season transfer operator which mimics the player transfers between teams and allows the component of better solutions being propagated among other solutions in certain iterations.



2. A review on the related terminology and the required background

In this section we shall have an overview on the keywords commonly related to team games, especially those terms which will be used metaphorically in LCA.

☆*Sport league*- A sports league is an organization that exists to provide a regulated competition for a number of people to compete in a specific sport. League is generally used to refer to competitions involving team sports, not individual sports. A league championship may be contested in a number of ways. Each team may play every other team a certain number of times in a round-robin tournament. In such a set-up, a team with the best record becomes champion, based on either a strict win-loss-tie system or on a points system where a certain number of points are awarded for a win, loss, or tie [11].

☆*Formations*- Normally each team has a playing style which can be realized during the game via team formation. A formation is a specific structure defining a distribution of players based on their positions within the field of play [12]. For example, the most common formations in soccer are variations of 4-4-2, 4-3-3, 3-2-3-2, 5-3-2 and 4-5-1. Different formations can be used depending on whether a team wishes to play more attacking or defensive. Every team pursues a *best formation* which is often related to the type of players available to the coach.

☆*Match analysis*- match analysis refers to the objective recording and examination of behavioural events occurring during competitions [13]. The main aim of match analysis when observing one's own team's performance is to identify *strengths* which can then be further built upon and *weaknesses* which suggest areas for improvement. Likewise, a coach analyzing opposition performance will use data to try to counter opposing strengths (*threats*) and exploit weaknesses (*opportunities*) [13]. An extremely important ingredient of the match analysis process is the presentation of feedback to players on their own or opponent's performance through video footage, match reconstructions and presentation of data. Feedback can and should be given pre-match, post-match or in the build up to next match [13].

Such kind of analysis is typically known as *strengths/weaknesses/opportunities/threats (SWOT)* analysis, which explicitly links internal (strengths and weaknesses) and external factors (opportunities and threats). Identification of SWOTs is essential because subsequent steps in the process of planning for achievement of the main objective may be derived from the SWOTs. The primary strength of SWOT analysis arises from matching specific internal and external factors and evaluating the multiple interrelationships involved. There are four basic categories of matches for which strategic alternatives can be considered [14]:

- *S/T matches* show the strengths in light of major threats from competitors. The team should use its strengths to avoid or defuse threats.
- *S/O matches* show the strengths and opportunities. The team should attempt to use its strengths to exploit opportunities.
- *W/T matches* show the weaknesses against existing threats. The team must attempt to minimize its weaknesses and avoid threats. Such strategies are generally defensive.
- *W/O matches* illustrate the weaknesses coupled with major opportunities. The team should try to overcome its weaknesses by taking advantage of opportunities.

Later, in section 3-3, we will use these strategies in the metaphorical "LCA match analysis process", wherein each artificial team devises a suitable team formation for its next match.

The SWOT analysis provides a structured approach to conduct the *gap analysis*. A gap is sometimes spoken of as "the space between where we are and where we want to be". When the process of identifying gaps includes a deep analysis of the factors that



have created the current state of the team, the groundwork has been laid for improvement planning. The gap analysis process can be used to ensure that the improvement process does not jump from identification of problem areas to proposed solutions without understanding the conditions that created the current state of the team.

☆*Transfer*-At the end of each season, teams review their performance over the past season and various changes may occur, e.g. changes in the coaching configuration, transfer of players or even changes in the managerial board. A transfer is the action taken whenever a player moves between clubs. It refers to the transferring of a player's registration from one team to another.

3. The League Championship Algorithm (LCA)

LCA is a population based algorithmic framework for global optimization over a continuous search space. A common feature among all population based algorithms like LCA is that they attempt to move a population of possible solutions to promising areas of the search space during seeking the optimum. Similar to most of population based algorithms, a set of L solutions in the search space, chosen a priori at random, form the initial population of LCA. Using the sporting terminology, "league" in LCA stands for "population". Like most of population based algorithms, LCA consists in evolving gradually the composition of the population in successive iterations, by maintaining the size of population constant. For the sake of consistency we may use "week" in place of "iteration". Each solution in the population is associated to one of L teams (L is an even number) and is interpreted as the team's current formation. Therefore, "team i " is matched to the " i^{th} member of the population" and a particular "formation" for team i is matched to the i^{th} "solution" in the population. Each solution in the population has a certain fitness value, which measures its degree of adaptation to the objective aimed. In LCA the "fitness value" can be interpreted as the "playing strength" along with the intended team formation. Almost in all population-based algorithms a succession of operators is applied to individuals in each iteration to generate the new solutions for the next iteration. The way in which a new solution associated to an LCA's team is generated is governed via imitating the match analysis process which is typically followed by coaches to design a suitable arrangement for their team. In section 3-3 we will explain how we make an analogy between the process of generating a new solution in LCA and the sport match analysis process. A broad group of population-based algorithms are Evolutionary Algorithms (EA) in which during iterations, the objective is to overall improve of the fitness of the individuals [15]. Such a result is obtained by simulating the selection mechanism, which governs the evolution of the living beings through supporting the survival of the fittest individuals, according to the Darwinian Theory. As a pseudo evolutionary algorithm, selection in LCA is a greedy selection which replaces the current best formation with a more productive team formation having a better playing strength. The algorithm terminates after a certain number of "seasons" (S) being passed in which each season comprises $L-1$ weeks (iterations), yielding $S \times (L-1)$ weeks of contests.

Now we idealize some characteristics of the regular championship environment to visualize the artificial championship modeled by LCA. Each of the following idealized rules will be used in the subsequent parts of the paper, where we provide a detailed report on different modules developed in LCA.

- Idealized rule 1. It is more likely that a team with better playing strength wins the game. The term "playing strength" refers to ability of one team to beat another team.
- Idealized rule 2. The outcome of a game is not foretellable given known the teams' playing strength perfectly.
- Idealized rule 3. The probability that team i beats team j is assumed equal from both teams point of view.
- Idealized rule 4. The outcome of the game is only win or loss. Tie outcome is not considered in the basic version of LCA (We will later break this assumption via inclusion of the tie outcome, when introducing other variants of the algorithm).



Idealized rule 5. When team i beats team j , any strength helped team i to win has a dual weakness caused team j to lose. In other words, any weakness is a lack of a particular strength. An implicit implication of this rule is that while the match outcome is imputed to chance, teams may not believe it technically.

Idealized rule 6. Teams only focus on their upcoming match without regards of the other future matches. Formation settings are done just based on the previous week events.

The basic steps of the League championship algorithm can be represented as a schematic flowchart shown in Fig 1. In practice, a representation must be chosen for the individuals in the population. In LCA, a team formation (solution) can be represented with a vector of size $1 \times n$ (n is the number of problem parameters or variables) of real numbers. Each element is associated to one of the players and represents the value of the corresponding variable of the problem. One can imagine that a change in the value of a variable reflects a change in the job of the relevant player in the new formation. Let $f(X = (x_1, x_2, \dots, x_n))$ be an n variable numerical function that should be minimized over the decision space defined as a subset of \mathbb{R}^n . A team formation (a potential solution) for team i at week t can be represented by $X_i^t = (x_{i1}^t, x_{i2}^t, \dots, x_{in}^t)$, with $f(X_i^t)$ indicating the fitness/function value resulted from X_i^t . Back to our terminology, this value is called the playing strength along with formation X_i^t . By $B_i^t = (b_{i1}^t, b_{i2}^t, \dots, b_{in}^t)$ we denote the best previously experienced formation by team i until week t , yielding the best playing strength value. To determine B_i^t , we employ a greedy selection between X_i^t and B_i^{t-1} based on the value of $f(X_i^t)$ and $f(B_i^{t-1})$.

In the subsequent subsections we will describe the main modules of LCA; especially the manner of generating the league schedule, determining the winner/loser; and setting up a new team formation.

3-1. Generating the league schedule

A common thread between all sport leagues is a structure that allows teams to compete against each other in a nonrandom order on a set schedule, usually called a *season*. Likewise in LCA, the first step necessary to simulate a championship environment is to schedule the matches in each season. A *single round-robin* schedule is utilized in which each team plays every other team once in each season. For a sport league composed of L teams, the single round-robin tournament requires $L(L-1)/2$ matches, because in each of $(L-1)$ rounds (weeks), $L/2$ matches will be run in parallel (if L is odd, there will be L rounds with $(L-1)/2$ matches, and one team have no game in that round).

The scheduling algorithm is simple and we illustrate it using a sport league composed of 8 teams ($L=8$). Let assign each team a number and pair them off in the first week (Fig 2a). Fig 2a implies that team 1 plays 8, 2 plays 7 and so on. For the second week, fix one team, say team 1, and rotate the others clockwise (Fig 2b). In this week, 1 plays 7, 8 plays 6 and so on. For the third week, once again rotate the order clockwise. So, 1 plays 6, 7 plays 5 and so on (Fig 2c). We continue this process until getting the initial state. The last week (week 7) schedule can be obtained from Fig 2d. If L is odd, a dummy team is added. In each week, the opponent of the dummy team does not play and gets rest.

It is worth to mention that the round-robin tournament problem can be modelled as an edge-coloring problem in a diagraph [16]. In LCA, the championship continues for S successive seasons in which the league schedule for each season is a single round-robin schedule, yielding $S \times (L-1)$ weeks of contests (if L is odd, there will be $S \times L$ weeks of contests). In our implementation of LCA we use the same schedule for all of the S seasons.



3-2. Determining the winner/loser

In a regular league system, teams compete on a weekly basis and their game outcome is determined in terms of win, loss or tie for each team. In this way, for example in soccer, each team is scored by 3 points for win, 0 for loss and 1 for tie. Disregarding the occasional crisis which may entrap even excellent teams in a continuum of abortive results, it is more likely that a more powerful team having a better playing strength beats the weaker one (idealized rule 1).

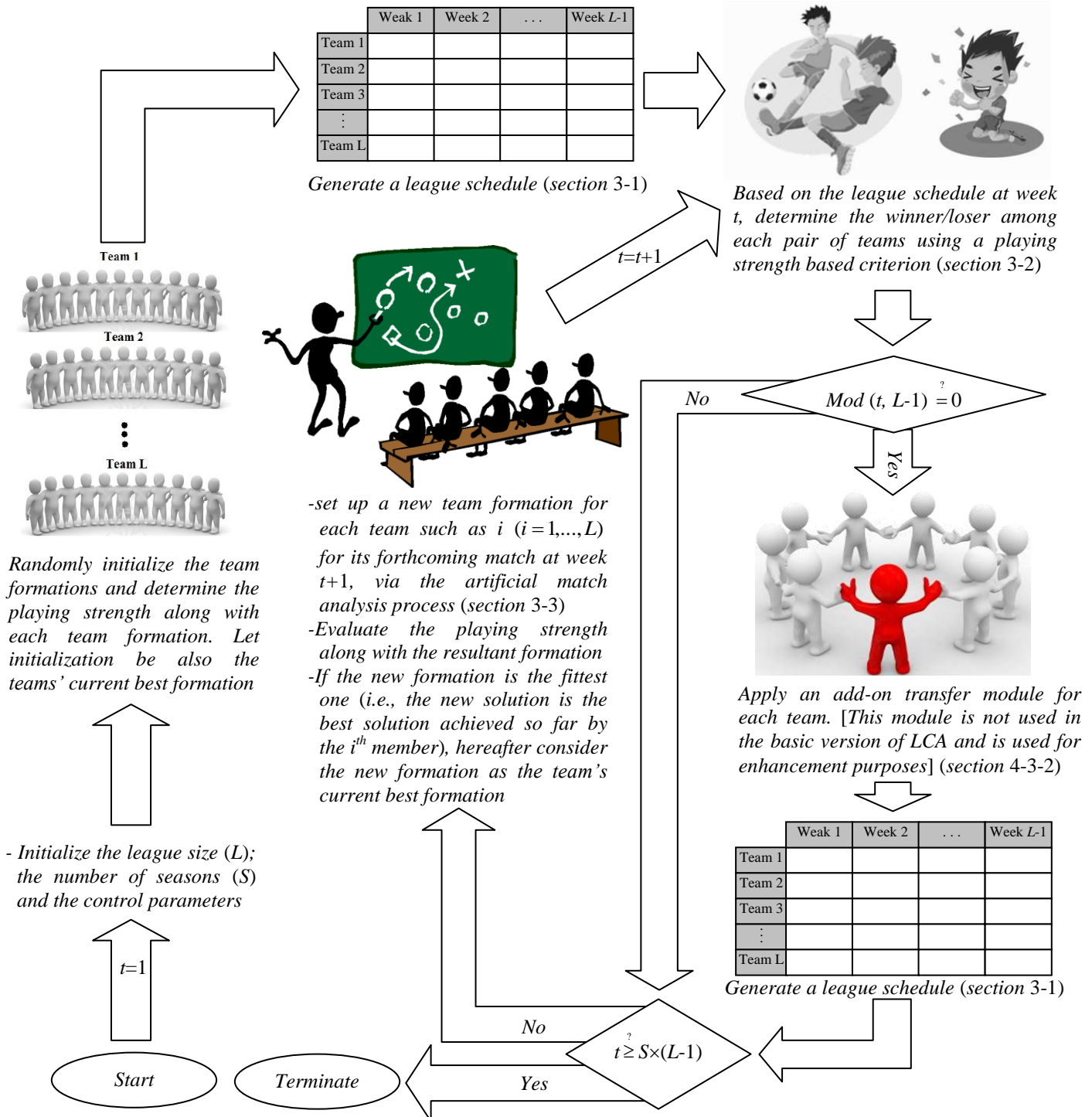


Fig 1. Flowchart of the league championship algorithm (LCA)



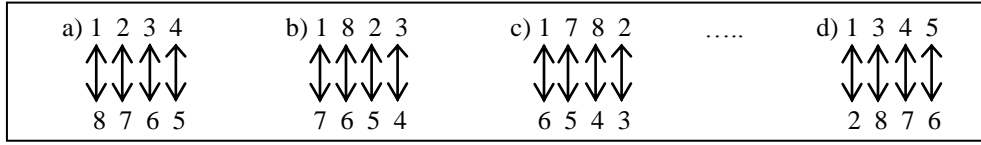


Fig 2. An illustrative example of the league scheduling algorithm.

Given an ideal league environment devoid of the influence of uninvited effects, we can assume a linear relationship between the playing strength of a team and the outcome of its game. Therefore, proportional to its playing strength, each team may have a chance of win. This conclusion comes from our second idealized rule.

Using the playing strength criterion, the winner/loser in LCA is recognized in a stochastic manner with this condition that the chance of win for a team is proportional to its degree of fit (recall that in the basic version of LCA there is no tie outcome). The degree of fit is proportional to the team's playing strength and is measured by means of the distance with an ideal reference point.

Let us consider teams i and j fighting at week t , with their formations X_i^t and X_j^t and playing strengths $f(X_i^t)$ and $f(X_j^t)$, respectively. Let p_i^t denotes the chance of team i to beat team j at week t (p_j^t can be defined accordingly). Let also \hat{f} be an ideal value (e.g., a lower bound on the optimal value). Based on the idealized rule 1 we can write

$$\frac{f(X_i^t) - \hat{f}}{f(X_j^t) - \hat{f}} = \frac{p_i^t}{p_j^t}. \quad (1)$$

Equation (1) implies that the expected chance of win for team i (or j) is proportional to the difference between its current playing strength and the ideal strength along with an ideal team. In equation (1) we assume that a better team can comply with more factors that an ideal team owns. Since teams are evaluated based on their distance with a common reference point, the ratio of distances can determine the winning portion for each team.

Based on the idealized rule 3 we can also write

$$p_i^t + p_j^t = 1. \quad (2)$$

From equations (1) and (2) we get

$$p_i^t = \frac{f(X_j^t) - \hat{f}}{f(X_j^t) + f(X_i^t) - 2\hat{f}}. \quad (3)$$

To determine the winner or loser, a random number in $[0,1]$ is generated; if it is less than or equal to p_i^t , team i wins and team j loses; otherwise j wins and i loses. Such a procedure for determining the winner/loser is consistent with idealized rules 2 and 4.

If $f(X_i^t)$ be arbitrarily closed to $f(X_j^t)$, then p_i^t can be arbitrarily closed to $\frac{1}{2}$. Moreover, if $f(X_i^t)$ becomes far greater than $f(X_j^t)$, namely $f(X_i^t) \gg f(X_j^t)$, then p_i^t approaches to 1. Since the value of \hat{f} may be unavailable in advance, we use from the best function value found so far (i.e., $\hat{f}^t = \min_{i=1, \dots, L} \{f(B_i^t)\}$).



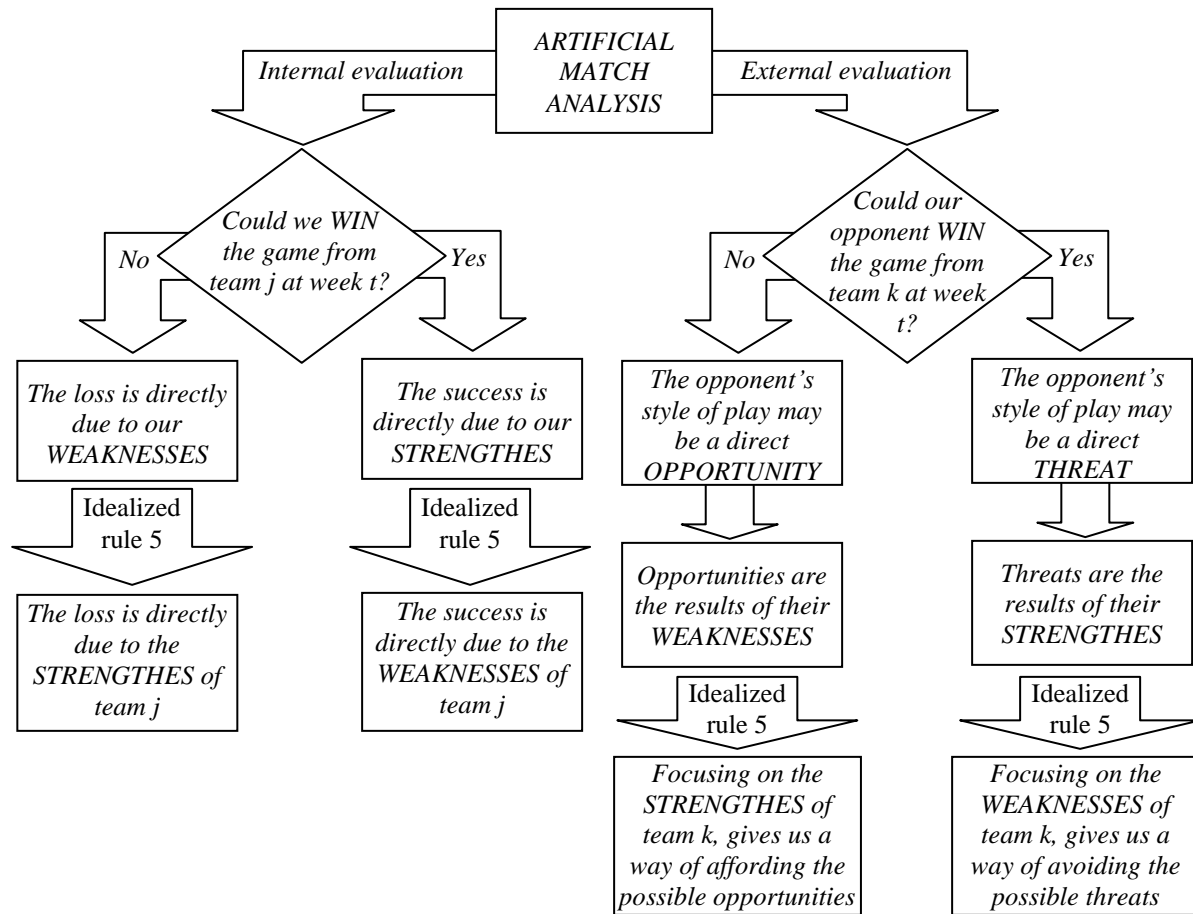


Fig 3. Artificial match analysis in LCA

3-3. Setting up a new team formation

Before any strategy is applied, it is important for a coach to evaluate the *strengths* and *weaknesses* of the individual members and the team as a whole. This will serve as a guide as to how to approach them and the kind of professional relationship that should be developed, which area to focus on, and how to teach the required game skills to enhance their performance. The analysis should also include the evaluation of *opportunities* and *threats* that comes along with the unique dynamics of the team. Strengths and weaknesses are often internal factors while opportunities and threats are external factors.

Likewise in LCA, the artificial analysis of the team's previous performance (at week t) is treated as internal evaluation (strengths/weaknesses) while analysis of the opponent's previous performance is accounted as external evaluation (opportunities/threats). Modelling an artificial match analysis for team (individual) i (to devise for a new team formation for week $t+1$), if it had won (lost) the artificial game from (to) team j at week t , then we assume that the success (loss) was directly due to the strengths (weaknesses) of team i or based on the idealized rule 5 of section 3, it was directly resulted by the weaknesses (strengths) of team j . Fig 3 (left branch) shows such a hypothetical internal evaluation for team i in a flowchart format. Now, based on the league schedule at week $t+1$, assume that the next match of team i is with team l . If team l had won (lost) the game from (to) team k at week t , then that success (loss) and the team formation behind it may be a direct threat (opportunity) for team i . Apparently, such a success (loss) has been achieved by means of some strengths (weaknesses). Focusing on the strengths (weaknesses) of team l , gives us an intuitive way to



avoid from the possible threats (to receive benefits from possible opportunities). Referring to idealized rule 5, we can focus on the weaknesses (strengths) of team k instead. Fig 3 (right branch) demonstrates the hypothetical external evaluation followed by team i .

Based on the previous week events (idealized rule 6 of section 3), the possible actions for team i derived from the artificial match analysis can be summarized in the hypothetical SWOT matrix of Fig 4. Given the basic S/T, S/O, W/T, W/O strategies explained earlier in section 2, Fig 4 demonstrates the conditions under which each of these metaphorical strategies is applicable for team i . For example, if team i had won and team l had lost, then it is reasonable that team i focuses on the strengths which made it capable to win. At the same time it should focus on the weaknesses that brought the loss for team l . These weaknesses may open opportunities for team i . Therefore, adopting an S/O type strategy would be a proper action for team i . The matrix presented in Fig 4 is a metaphorical demonstration of the SWOT matrix which is typically used in planning [14].




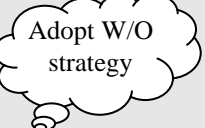
	 <i>i had won</i> <i>l had won</i> Focusing on ...	 <i>i had won</i> <i>l had lost</i> Focusing on ...	 <i>i had lost</i> <i>l had won</i> Focusing on ...	 <i>i had lost</i> <i>l had lost</i> Focusing on ...
S	own strengths (or weaknesses of j)	own strengths (or weaknesses of j)	–	–
W	–	–	own weaknesses (or strengths of j)	own weaknesses (or strengths of j)
O	–	weaknesses of l (or strengths of k)	–	weaknesses of l (or strengths of k)
T	strengths of l (or weaknesses of k)	–	strengths of l (or weaknesses of k)	–

Fig 4. Hypothetical SWOT matrix derived from the artificial match analysis.

The above analysis is carried out by all participants during week $t+1$ to plan for a suitable team formation for their next match at the end of week $t+1$. After adopting a suitable focus strategy with the aid of the artificial SWOT matrix of Fig 4, now teams should try to fill their gaps. For example, assume that team i has lost the game to team j and during match analysis it has been detected that the reason was for the weakness in a man to man defence (which allowed counter attacks by team j). Therefore, there is a gap between the current penetrable defensive state and the state which ensures a man to man pressure defence.

Let us introduce the following indices:

l = Index of the team that will play with team i ($i = 1, \dots, L$) at week $t+1$ based on the league schedule.

j = Index of the team that has played with team i ($i = 1, \dots, L$) at week t based on the league schedule.

k = Index of the team that has played with team l at week t based on the league schedule.



Let X'_i , X'_j and X'_k be the team formations associated to teams i, j and k at week t , respectively. By “ $X'_k - X'_i$ ” we address the gap between the playing style of team i and team k , sensed via “focusing on the strengths of team k ”. In this situation, team k has won the game from team l and to beat l , it is reasonable that team i devises for a playing style almost similar to that was adopted by team k at week t (for example, playing counter attacking or high pressure defence). In a similar way we can interpret “ $X'_i - X'_k$ ” when “focusing on the weaknesses of team k ”. Here, it may be sensible to avoid a playing style rather similar to that was adopted by team k (for example, avoid playing counter attacking or high pressure defence). We can interpret “ $X'_j - X'_i$ ” or “ $X'_i - X'_j$ ” in a similar manner.

Given the fact that usually teams play based on their current best formation (found it suitable over the time) while preparing the required changes recommended by the match analysis, the new formation $X_i^{t+1} = (x_{i1}^{t+1}, x_{i2}^{t+1}, \dots, x_{im}^{t+1})$ for team i ($i = 1, \dots, L$) at week $t+1$ can be set up by one of the following equations.

If i had won and l had won too, then the new formation is generated based on the adaptation of S/T strategy

$$(S/T \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (x_{kd}^t - x_{id}^t) + \psi_1 r_{2id} (x_{id}^t - x_{jd}^t)) \quad \forall d = 1, \dots, n \quad (4)$$

Else if i had won and l had lost, then the new formation is generated based on the adaptation of S/O strategy

$$(S/O \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (x_{kd}^t - x_{id}^t) + \psi_1 r_{2id} (x_{id}^t - x_{jd}^t)) \quad \forall d = 1, \dots, n \quad (5)$$

Else if i had lost and l had won, then the new formation is generated based on the adaptation of W/T strategy

$$(W/T \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (x_{id}^t - x_{kd}^t) + \psi_2 r_{2id} (x_{jd}^t - x_{id}^t)) \quad \forall d = 1, \dots, n \quad (6)$$

Else if i had lost and l had lost too, then the new formation is generated based on the adaptation of W/O strategy

$$(W/O \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (x_{kd}^t - x_{id}^t) + \psi_2 r_{2id} (x_{jd}^t - x_{id}^t)) \quad \forall d = 1, \dots, n \quad (7)$$

End if

In the above equations, d is the variable or dimension index. r_{1id} and r_{2id} are uniform random numbers in $[0,1]$. ψ_1 and ψ_2 are coefficients used to scale the contribution of “retreat” or “approach” components, respectively. Note that the difference sign in parenthesis results in acceleration toward winner or retreat from loser.

In equations (4) to (7), y_{id}^t is a binary change variable which indicates whether x_{id}^{t+1} differs from b_{id}^t or not. Only $y_{id}^t = 1$ allows for difference. Let us define $Y_i^t = (y_{i1}^t, y_{i2}^t, \dots, y_{im}^t)$ as the binary change array in which the number of ones is equal to q_i^t . It is not usual that coaches do changes in all or many aspects of their team. Normally a small number of changes are recommended. By analogy, it is sensible that the number of changes made in B_i^t (i.e., the value of q_i^t) be small. To simulate the number of changes, we use a truncated geometric probability distribution [17]. Using a truncated geometric distribution, we can set the number of changes dynamically with more emphasis given to the smaller rate of changes. The following formula simulates the random number of changes made in B_i^t to get the new team formation X_i^{t+1} (see the appendix).

$$q_i^t = \left\lceil \frac{\ln(1 - (1 - (1 - p_c)^{n - q_0 + 1})r)}{\ln(1 - p_c)} \right\rceil + q_0 - 1 \quad : q_i^t \in \{q_0, q_0 + 1, \dots, n\} \quad (8)$$



Where r is a random number in $[0,1]$ and $p_c < 1, p_c \neq 0$ is a control parameter. If $p_c < 0$, then the situation is reversed and the more negative the value of p_c , the more emphasis is given to the greater rate of changes. q_0 is the least number of changes realized during the artificial match analysis. We assume that the number of changes made in the best formation is at least one, (i.e., $q_0 = 1$). Typically, p_c is known as the probability of success in the truncated geometric distribution. The greater the value of p_c , the smaller number of changes are recommended. After simulating the number of changes by (8), q_i' number of elements are selected randomly from B_i' and their value changes according to one of equations (4) to (7).

Example: The step-wise procedure for the implementation of the first iteration of LCA is given as follows. For demonstration of the procedure, the Rastrigin multimodal function is considered in 3 dimensions. We assume that the league size (L) is 4 and $\psi_1 = \psi_2 = 1$.

1) Randomly initialize the team formations according to the league size and the number of variables and determine the playing strength along with each team formation. Let initialization be also the teams' current best formation.

$$\text{Initial team formations} = \begin{bmatrix} \text{Team 1} \\ \text{Team 2} \\ \text{Team 3} \\ \text{Team 4} \end{bmatrix} = \begin{bmatrix} X_1^1 \\ X_2^1 \\ X_3^1 \\ X_4^1 \end{bmatrix} = \begin{bmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1n}^1 \\ x_{21}^1 & x_{22}^1 & \cdots & x_{2n}^1 \\ \vdots & \vdots & \vdots & \vdots \\ x_{L1}^1 & x_{L2}^1 & \cdots & x_{Ln}^1 \end{bmatrix} = \begin{bmatrix} B_1^1 \\ B_2^1 \\ B_3^1 \\ B_4^1 \end{bmatrix} = \begin{bmatrix} b_{11}^1 & b_{12}^1 & \cdots & b_{1n}^1 \\ b_{21}^1 & b_{22}^1 & \cdots & b_{2n}^1 \\ \vdots & \vdots & \vdots & \vdots \\ b_{L1}^1 & b_{L2}^1 & \cdots & b_{Ln}^1 \end{bmatrix} = \begin{bmatrix} 1.5574 & 1.7873 & 1.5547 \\ -4.6428 & 2.5774 & -3.2881 \\ 3.4912 & 2.4313 & 2.0604 \\ 4.3399 & -1.0777 & -4.6816 \end{bmatrix}$$

$$\text{The corresponding playing strength (objective function) value} = \begin{bmatrix} f(X_1^1) \\ f(X_2^1) \\ f(X_3^1) \\ f(X_4^1) \end{bmatrix} = \begin{bmatrix} 54.4821 \\ 86.4584 \\ 62.1273 \\ 72.6008 \end{bmatrix} \Rightarrow \hat{f} = \min_{i=1,\dots,4} \{f(B_i^1)\} = 54.4821$$

2) Generate a league schedule.

	Week 1	Week 2	Week 3
Team 1	Team 4	Team 3	Team 2
Team 2	Team 3	Team 4	Team 1
Team 3	Team 2	Team 1	Team 4
Team 4	Team 1	Team 2	Team 3

3) Based on the league schedule at week 1, determine the winner/loser among each pair of teams using Equation (3).

$$\left. \begin{aligned} p_1^1 &= \frac{f(X_4^1) - \hat{f}}{f(X_4^1) + f(X_1^1) - 2\hat{f}} = \frac{72.5008 - 54.4821}{72.5008 + 54.4821 - 2 \times 54.4821} = 1 \\ p_4^1 &= \frac{f(X_1^1) - \hat{f}}{f(X_4^1) + f(X_1^1) - 2\hat{f}} = \frac{54.4821 - 54.4821}{72.5008 + 54.4821 - 2 \times 54.4821} = 0 \end{aligned} \right\} \Rightarrow \begin{cases} \text{Team 1 is winner} \\ \text{Team 4 is loser} \end{cases}$$

$$\left. \begin{aligned} p_2^1 &= \frac{f(X_3^1) - \hat{f}}{f(X_3^1) + f(X_2^1) - 2\hat{f}} = \frac{62.1273 - 54.4821}{62.1273 + 86.4584 - 2 \times 54.4821} = 0.193 \\ p_3^1 &= \frac{f(X_2^1) - \hat{f}}{f(X_3^1) + f(X_2^1) - 2\hat{f}} = \frac{86.4584 - 54.4821}{62.1273 + 86.4584 - 2 \times 54.4821} = 0.807 \end{aligned} \right\} \xrightarrow{r=0.159 < 0.193} \begin{cases} \text{Team 2 is winner} \\ \text{Team 3 is loser} \end{cases}$$



4) Assume that $q_1^1 = q_3^1 = q_4^1 = 1$, $q_2^1 = 2$ and therefore assume that $Y_1^1 = (0,1,0)$, $Y_2^1 = (1,1,0)$, $Y_3^1 = (1,0,0)$, $Y_4^1 = (0,0,1)$. To generate q_1^1, \dots, q_4^1 we use from Equation (8). Set up a new team formation for each team for its forthcoming match at week 2, via the artificial match analysis process and apply the greedy selection to find the team's best formation as follows (the value of r_{1id} and r_{2id} is generated randomly):

$$i = 1 \xrightarrow[\substack{j=4, l=3, k=2, r_{112}=0.225, r_{212}=0.512}]{\text{Team 1 has won and Team 3 has lost. Use S/O strategy}} \begin{cases} x_{11}^2 = b_{11}^1 = 1.5574 \\ x_{12}^2 = b_{12}^1 + y_{12}^1(\psi_2 r_{112}(x_{22}^1 - x_{12}^1) + \psi_1 r_{212}(x_{12}^1 - x_{42}^1)) = \\ \quad 1.7873 + 0.225(2.5774 - 1.7873) + 0.512(1.7873 + 1.0777) = 3.4319 \\ x_{13}^2 = b_{13}^1 = 1.5547 \end{cases}$$

$$i = 2 \xrightarrow[\substack{j=3, l=4, k=1, r_{121}=0.124, r_{221}=0.954, r_{122}=0.765, r_{222}=0.034}]{\text{Team 2 has won and Team 4 has lost. Use S/O strategy}} \begin{cases} x_{21}^2 = b_{21}^1 + y_{21}^1(\psi_2 r_{121}(x_{11}^1 - x_{21}^1) + \psi_1 r_{221}(x_{21}^1 - x_{31}^1)) = \\ \quad -4.6428 + 0.124(1.5574 + 4.6428) + 0.954(-4.6428 - 3.4912) = -11.6338 \\ x_{22}^2 = b_{22}^1 + y_{22}^1(\psi_2 r_{122}(x_{12}^1 - x_{22}^1) + \psi_1 r_{222}(x_{22}^1 - x_{32}^1)) = \\ \quad 2.5774 + 0.765(1.7873 - 2.5774) + 0.034(2.5774 - 2.4313) = 1.9779 \\ x_{23}^2 = b_{23}^1 = -3.2881 \end{cases}$$

$$i = 3 \xrightarrow[\substack{j=2, l=1, k=4, r_{131}=0.478, r_{231}=0.201}]{\text{Team 3 has lost and Team 1 has won. Use W/T strategy}} \begin{cases} x_{31}^2 = b_{31}^1 + y_{31}^1(\psi_1 r_{131}(x_{31}^1 - x_{41}^1) + \psi_2 r_{231}(x_{21}^1 - x_{31}^1)) = \\ \quad 3.4912 + 0.478(3.4912 - 4.3399) + 0.201(-4.6428 - 3.4912) = 1.4505 \\ x_{32}^2 = b_{32}^1 = 2.4313 \\ x_{33}^2 = b_{33}^1 = 2.0604 \end{cases}$$

$$i = 4 \xrightarrow[\substack{j=1, l=2, k=3, r_{143}=0.871, r_{243}=0.367}]{\text{Team 4 has lost and Team 2 has won. Use W/T strategy}} \begin{cases} x_{41}^2 = b_{41}^1 = 4.3399 \\ x_{42}^2 = b_{42}^1 = -1.0777 \\ x_{43}^2 = b_{43}^1 + y_{43}^1(\psi_1 r_{143}(x_{43}^1 - x_{33}^1) + \psi_2 r_{243}(x_{13}^1 - x_{43}^1)) = \\ \quad -4.6816 + 0.871(-4.6816 - 2.0604) + 0.367(1.5547 + 4.6816) = -8.2651 \end{cases}$$

$$\text{New team formations for week 2 contests} = \begin{bmatrix} \text{Team 1} \\ \text{Team 2} \\ \text{Team 3} \\ \text{Team 4} \end{bmatrix} = \begin{bmatrix} X_1^2 \\ X_2^2 \\ X_3^2 \\ X_4^2 \end{bmatrix} = \begin{bmatrix} 1.5574 & 3.4319 & 1.5547 \\ -11.6338 & 1.9779 & -3.2881 \\ 1.4505 & 2.4313 & 2.0604 \\ 4.3399 & -1.0777 & -8.2651 \end{bmatrix}$$

$$\text{The corresponding playing strength (objective function) value} = \begin{bmatrix} f(X_1^2) \\ f(X_2^2) \\ f(X_3^2) \\ f(X_4^2) \end{bmatrix} = \begin{bmatrix} 74.4908 \\ 179.2058 \\ 51.5749 \\ 115.7765 \end{bmatrix}$$

$$\text{Greedy selection phase:} \begin{cases} f(B_1^1) < f(X_1^2) \Rightarrow B_1^2 \leftarrow B_1^1 \\ f(B_2^1) < f(X_2^2) \Rightarrow B_2^2 \leftarrow B_2^1 \\ f(B_3^1) > f(X_3^2) \Rightarrow B_3^2 \leftarrow X_3^2 \\ f(B_4^1) < f(X_4^2) \Rightarrow B_4^2 \leftarrow B_4^1 \end{cases} \Rightarrow \begin{bmatrix} B_1^2 \\ B_2^2 \\ B_3^2 \\ B_4^2 \end{bmatrix} = \begin{bmatrix} 1.5574 & 1.7873 & 1.5547 \\ -4.6428 & 2.5774 & -3.2881 \\ 1.4505 & 2.4313 & 2.0604 \\ 4.3399 & -1.0777 & -4.6816 \end{bmatrix}$$



$$\Rightarrow \begin{bmatrix} f(B_1^2) \\ f(B_2^2) \\ f(B_3^2) \\ f(B_4^2) \end{bmatrix} = \begin{bmatrix} 54.4821 \\ 86.4584 \\ 51.5749 \\ 72.6008 \end{bmatrix} \Rightarrow \hat{f} = \min_{i=1,\dots,4} \{f(B_i^1)\} = 51.5749$$

5) The process is stopped if the maximum number of function evaluations achieved; otherwise repeat from step 3).

Equations (4) to (7) use the teams' most recent formations as a basis to determine the new formation X_i^{t+1} . Let us use the notation "LCA/recent" to address such a variant of LCA. It is also possible to introduce another variant of LCA through developing an alternative set of equations for generating the new team formations. Instead of using the most recent formations in the differential operations in equations (4) to (7), we can use the current best formations. This means replacing x^t with b^t in equations (4) to (7). Therefore, an alternative system of equations can be introduced by equations (9) to (12), respectively.

$$(S/T \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_1 r_{2id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (9)$$

$$(S/O \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (b_{kd}^t - b_{id}^t) + \psi_1 r_{2id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (10)$$

$$(W/T \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (11)$$

$$(W/O \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (b_{kd}^t - b_{id}^t) + \psi_2 r_{2id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (12)$$

Using our twofold notation, the new variant is introduced by "LCA/best". The idea behind introducing this variant is closely related to the fact that generally, changes in the legacy formations are minor and therefore coaches are able to anticipate the playing strategy of their opponents relying on the knowledge they have acquired over the time from the opponents' best style. In order to accelerate the convergence process, when a team formation replaces its best formation in LCA/best, it is allowed that the new solution, which is a better solution, contributes in creation of the other two solutions in the current generation. In this way, a promising solution does not need to wait for the next generation to share its components.

Before finishing this section, it is worthy of note that in LCA diversification is controlled by allowing "retreat" from a solution and also by coefficient ψ_1 , while intensification is implicitly controlled by permitting "approach" to a solution and by coefficient ψ_2 . Moreover, the greedy selection employed to update the current best formations affects the intensification ability of the algorithm.

4. Experiments

In order to evaluate how well LCA performs on finding the global minimum of numerical functions, 5 test functions are benchmarked from literature [18]. All functions are optimized in the absence of any constraint, with the exception of the range constraints. With the aid of these functions we: 1) compare the performance of LCA with other algorithms, 2) try to validate the system of updating equations and learning strategies developed in LCA, 3) examine the effectiveness of simulating win/loss/tie system in place of win/loss system modeled in the basic version of LCA and 4) evaluate the impact of introducing a "transfer" like add-on module on the convergence speed of the algorithm.

As we already described, the set of test functions includes 5 functions. The first function f_1 is the 2-dimensional Schaffer F6 function. Function f_2 is the unimodal Sphere function which is relatively an easy function. Function f_3 is the Griewank function and is strongly multimodal. In this function the number of local optima increases with the dimensionality. Function f_4 is the Rastrigin function.



The cosine terms in this function produce many local minima. Therefore, this function is multimodal and optimization algorithms can be trapped easily in a local optimum. Function f_5 is the Rosenbrock function which is repeatedly used to test the performance of the optimization algorithms. The global minimum in this function is inside a long narrow valley and this makes the convergence toward the global minimum difficult. The minimum value of these functions is equal to zero.

Krink et al. [18] have investigated the performance of EA, differential evolution (DE) and particle swarm optimization (PSO) algorithms on these functions. Results obtained by these algorithms are directly adopted from Krink et al. [18] and recorded in Table 1. The first value reported in each cell of Table 1 is the mean of best function values obtained in 30 independent runs of the algorithms. The second value is the standard deviation of the best function values. The maximum number of functions evaluated by the algorithms was 100,000 for the low dimensional Schaffer F6 (2-dimensional) and Sphere (5-dimensional) functions, and 500,000 for 50-dimensional Griewank, Rastrigin, and Rosenbrock functions, respectively [18].

In order to equalize the maximum number of function evaluations, we allow only 100,000 evaluations for the first two functions and 500,000 evaluations for the other three functions in each run of LCA. Each experiment is repeated 30 times by both LCA/recent and LCA/best with different initial random seeds and the mean and standard deviation of the best function values are recorded in Table 1. Values less than $1E-12$ are assumed to be 0. However, we do not know the level of precision used by Krink et al. [18] to round their results. The following setting for control parameters is used in both versions of LCA: $\psi_1 = 0.2$; $\psi_2 = 1$; $p_c = 0.5$; and $L = 60$.

As seen from the results, both LCA/recent and LCA/best show a competitive performance. Without considering its relatively poor performance on the Rosenbrock function, DE finds the global optimum of all functions. But, PSO exhibits a poor performance almost on all functions. Unlike its good performance on low dimensional functions, the performance of EA deteriorates on the high dimensional functions. On the Rosenbrock function which is the most challenging function, both versions of LCA could reach the best mean values far less than the mean values reported by other algorithms. However, for Schaffer F6 function, under the above setting of parameters, LCA cannot ensure reaching the global optimum ($1E-12$). Though, we will show that introducing a “Transfer” like add-on module in LCA can give it a fast global optimum finding ability, such that it always hits the global optimum of the Schaffer F6 function with a few number of function evaluations. On the Sphere, Griewank and Rastrigin functions, the performance of LCA is consistent and it always finds the global optimum.

Table 1. Results obtained by EA [18], DE [18], PSO [18] and LCA on the basic functions

Function	EA	DE	PSO	LCA/recent	LCA/best
f_1	0 ± 0	0 ± 0	0.00453 ± 0.00090	$1.33E-7 \pm 1.63E-7$	$2.23E-9 \pm 1.15E-8$
f_2	0 ± 0	0 ± 0	$2.51130E-8 \pm 0$	0 ± 0	0 ± 0
f_3	0.00624 ± 0.00138	0 ± 0	1.54900 ± 0.06695	0 ± 0	0 ± 0
f_4	32.6679 ± 1.94017	0 ± 0	13.1162 ± 1.44815	0 ± 0	0 ± 0
f_5	79.8180 ± 10.4477	35.3176 ± 0.27444	5142.45 ± 2929.47	0.06 ± 0.15	0.06 ± 0.10

4-1. Effect of updating equations

In order to examine that whether each of S/T , S/O , W/T and W/O updating equations has a significant effect on the performance of LCA, we sequentially omit the effect that each equation might have on the evolution of the solutions. This can be done temporarily by



replacing the target equation with something like $x_{id}^{t+1} = b'_{id}$ in LCA/best. In this way, the target equation will not contribute in the generation of the new solutions. Table 2 summarizes the obtained results. In this table, a third part is also added to our twofold notation to display further information on the algorithm. For example “LCA/best/omitting S/T equation” indicates that equation (9) has been omitted from the system of updating equations.

From the results of Table 2 it can be inferred that the best performance is achieved when all of the four updating equations, typically used in LCA/best, are preserved. In other words, the effect of each equation is significant on at least one function. The progress of the mean of best function values is visualized in Figs 5 to 9 for each function under omission of each equation. An important point that can be extracted from these figures is that the curve associated to LCA/best always lies on the left side, underneath of other curves, which indicates a faster convergence provided via inclusion of all equations. As another observation, there is always one function for which the progress curve associated to an omitted equation lies on the right side, above of the other curves, exhibiting a slower convergence. For example: on f_1 the blue-colored curve lies upside; on f_3 the orange-colored curve lies upside; on f_4 the pink-colored curve lies upside; and on f_5 the green-colored curve lies upside. As a consequence, all updating equations have a significant contribution in the performance of the algorithm.

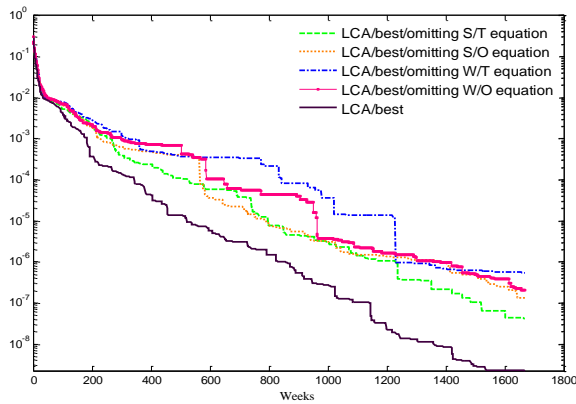


Fig 5. Progress of the mean of best function values for Shafer F6 function under imperfect system of LCA equations.

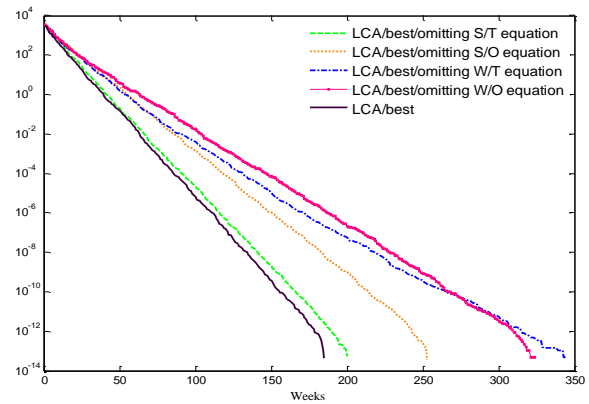


Fig 6. Progress of the mean of best function values for Sphere function under imperfect system of LCA equations.

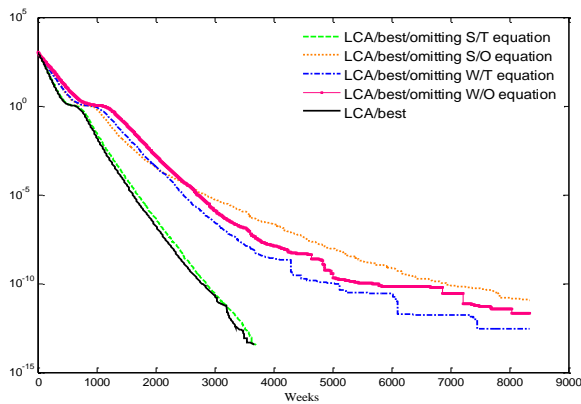


Fig 7. Progress of the mean of best function values for Griewank function under imperfect system of LCA equations.

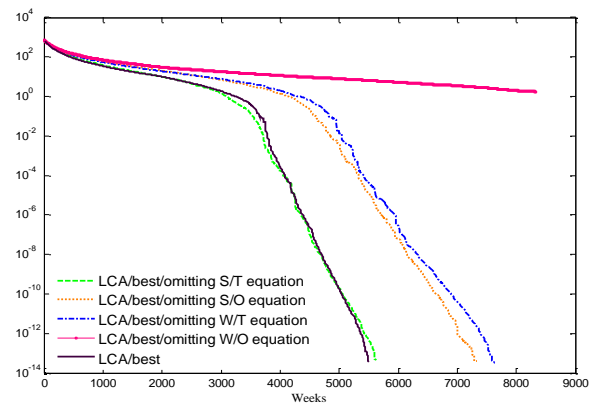


Fig 8. Progress of the mean of best function values for Rastrigin function under imperfect system of LCA equations.



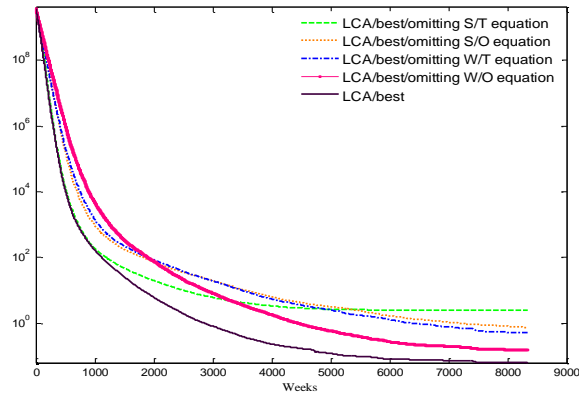


Fig 9. Progress of the mean of best function values for Rosenbrock function under imperfect system of LCA equations.

Table 2. Results obtained under imperfect system of LCA equations

Function	LCA/best/omitting S/T equation	LCA/best/omitting S/O equation	LCA/best/omitting W/T equation	LCA/best/omitting W/O equation	LCA/best
f_1	4.26E-8 ± 1.43E-7 14	1.31E-7 ± 2.70E-7 5	5.47E-7 ± 2.21E-6 2	2.09E-7 ± 3.88E-7 0	2.23E-9 ± 1.15E-8 15
f_2	0 ± 0 30	0 ± 0 30	0 ± 0 30	0 ± 0 30	0 ± 0 30
f_3	0 ± 0 30	1.13E-11 ± 4.71E-11 25	0 ± 1.46E-12 29	2.04E-12 ± 1.085E-11 28	0 ± 0 30
f_4	0 ± 0 30	0 ± 0 30	0 ± 0 30	1.72 ± 0.76 0	0 ± 0 30
f_5	2.44 ± 3.61 0	0.76 ± 0.85 0	0.51 ± 0.93 0	0.15 ± 0.36 0	0.06 ± 0.10 0

4-2. Effect of using alternative learning strategies in the artificial post-match analysis

Another analysis which is worth to investigate is to validate the system of equations developed in LCA, in terms of the type of learning strategy followed in the artificial post-match analysis. In equations (4) to (7) and (9) to (12) we assumed that the sources of learning from the artificial post-match analysis are the team's previous game and the opponent's previous game. In this section, we examine the situations in which there is only one source of learning, i.e., learning only from the team's previous game (internal learning) or learning only from the opponent's previous game (external learning). Results obtained by LCA/best under each of these learning scenarios are compared with the results obtained from the basic algorithm in Table 3.

In "LCA/best/Learning only from the team's previous game" there are two updating equations working based on the team's previous state of win or loss.

If i had won, then

$$(S \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (13)$$

Else if i had lost, then

$$(W \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{2id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (14)$$

End if



In “LCA/best/Learning only from the opponent’s previous game” there are again two updating equations working based on the opponent’s previous state of win or loss.

If I had won, then

$$(T \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{id} (b_{id}^t - b_{kd}^t)) \quad \forall d = 1, \dots, n \quad (15)$$

Else if I had lost, then

$$(O \text{ equation}): x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{id} (b_{kd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (16)$$

End if

From the progress plots in Figs 10 to 14, we can see that both of internal and external learning used in LCA/best are necessary to guarantee a faster convergence. In the absence of any external source of learning in “LCA/best/Learning only from the team’s previous game”, although the performance on f_1 is better than LCA/best (see Table 3), it is deteriorated on f_5 . For f_2, f_3 and f_4 , the performances are comparable. However, plots show that LCA/best converges faster and finds the global minimum of these functions with a fewer number of iterations.

Results show that relying only on the external source of learning results in a relatively poor performance. Almost on all functions, “LCA/best/learning only from the opponent’s previous game” reports a mean of best function values far greater than those reported by the other two algorithms.

It is interesting to note that these empirical results are in accordance with the business reality. In business strategy there are two schools of thought, the “environmental (external)” and the “resource based (internal)”. Through 1970s and 80s, the dominant school was the environmental school which dictates that a firm should analyse the forces present within the environment in order to assess the profit potential of the industry, and then design a strategy that aligns the firm to the environment. Nevertheless, above average performance is more likely to be the result of core capabilities inherent in a firm’s resources (internal view) than its competitive positioning in its industry (external view) [20].

4-3. Toward speeding up the convergence of LCA

In this section we are going to model more concepts of the real world league championships to introduce some new add-on parts into LCA. The main concern of such a modeling is to possibly enhance the convergence of the algorithm toward the global minima. In particular we are interested to allow that the “tie” outcome be also achievable beside the strictly win/loss outcome during “determining the winner/loser” phase. We also introduce a “transfer” like operator which mimics the end season movement of players between clubs. With this operator, the components of the better solutions are inserted into the weaker ones with the aim of possibly enhancing the quality of the whole population.

Table 3. Results obtained under different post-match learning strategies

Function	LCA/best/Learning only from the team’s previous game		LCA/best/Learning only from the opponent’s previous game		LCA/best (Learning from both team’s and opponent’s previous game)	
f_1	3.93E-13 ± 1.65E-12	28	3.25E-4 ± 1.77E-3	11	2.23E-9 ± 1.15E-8	15
f_2	0 ± 0	30	0 ± 0	30	0 ± 0	30
f_3	0 ± 0	30	1.71E-12 ± 6.60E-11	28	0 ± 0	30
f_4	0 ± 0	30	5.46E-10 ± 1.85E-9	8	0 ± 0	30
f_5	0.67 ± 0.77	0	1.42 ± 1.01	0	0.06 ± 0.10	0



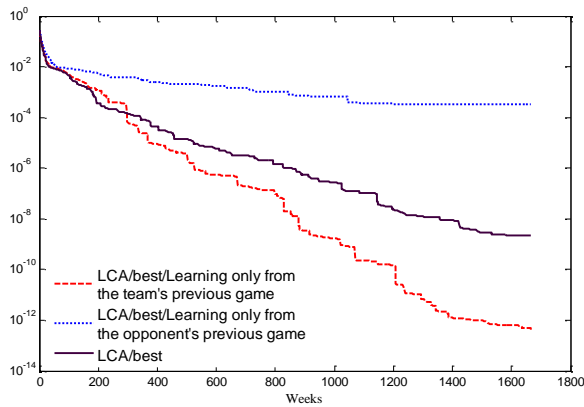


Fig 10. Progress of the mean of best function values for Shafer F6 function under different post-match learning strategies.

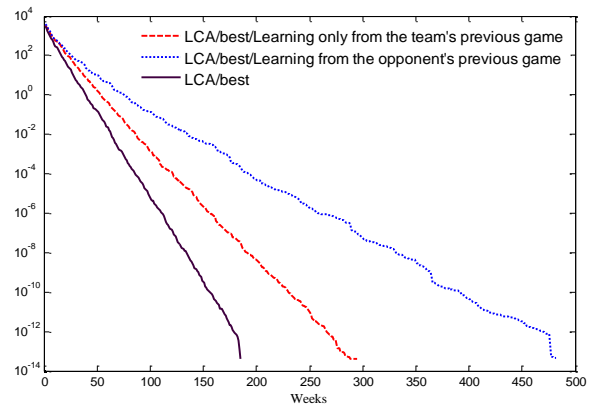


Fig 11. Progress of the mean of best function values for Sphere function under different post-match learning strategies.

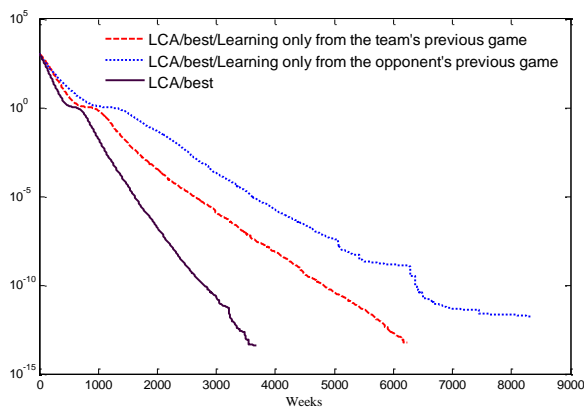


Fig 12. Progress of the mean of best function values for Griewank function under different post-match learning strategies.

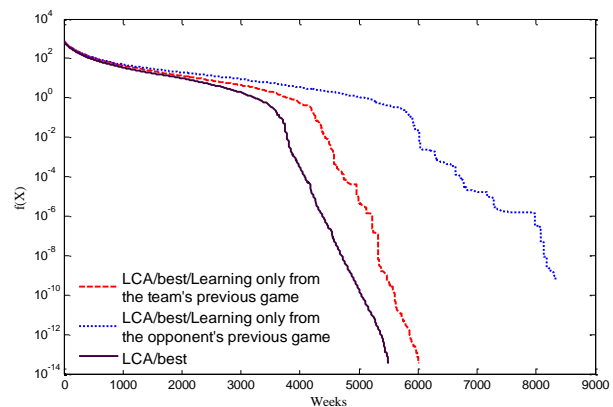


Fig 13. Progress of the mean of best function values for Rastrigin function under different post-match learning strategies.

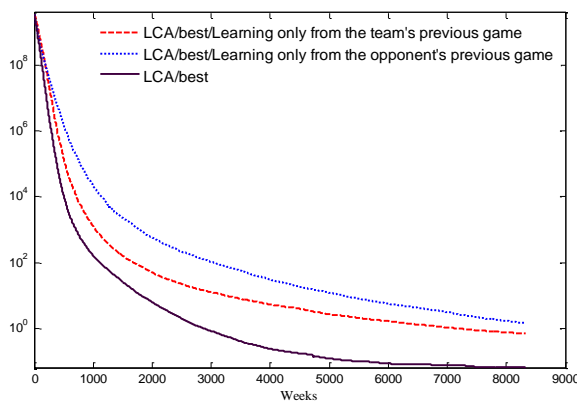


Fig 14. Progress of the mean of best function values for Rosenbrock function under different post-match learning strategies.

4-3-1. Inclusion of the tie outcome

To simulate a tie outcome, following section 3-2, two random numbers in $[0,1]$ are generated; if both are less than or equal to p_i' (see equation (3)), then team i wins and team j loses; otherwise if both are greater than p_i' , then j wins and i loses. If one is smaller than



p_i^t and the other one is greater than p_i^t , then the outcome of the game is considered as a tie. In the basic versions of LCA, we could decide to either come close to or retreat from the opponents' style of play (team formation), based on the win or loss states, during the gap analysis phase. However, in case of a tie there is no direct perception to select between getting approach or retreat. Here, we consider the following interpretations from a tie outcome during the artificial match analysis.

- *Tie outcome is interpreted as a consequent of strengths/opportunities and weaknesses/threats* (LCA/best/win-loss-tie 1). In this situation there are nine possible conditions where the first four conditions are the same as those used in LCA/best (equations (9) to (12)) and the other five conditions together with their relevant updating equation are as follows.

Else if i had won and l had tied

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (b_{kd}^t - b_{id}^t) + \psi_1 r_{3id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (17)$$

Else if i had tied and l had won

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_1 r_{2id} (b_{id}^t - b_{jd}^t) + \psi_2 r_{3id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (18)$$

Else if i had tied and l had tied too,

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (b_{kd}^t - b_{id}^t) + \psi_1 r_{3id} (b_{id}^t - b_{jd}^t) + \psi_2 r_{4id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (19)$$

Else if i had lost and l had tied

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (b_{kd}^t - b_{id}^t) + \psi_2 r_{3id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (20)$$

Else if i had tied and l had lost

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (b_{kd}^t - b_{id}^t) + \psi_1 r_{2id} (b_{id}^t - b_{jd}^t) + \psi_2 r_{3id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (21)$$

For example in (17), the term $\psi_1 r_{1id} (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (b_{kd}^t - b_{id}^t)$ which is associated to “l had tied”, construes that team i should focus on the possible opportunities via adopting a style of play close to the style of team k, and at the same time should avoid some playing strategies followed by k to neutralize the future threats. The consequent of such a conflict is considered as the points sensed from analyzing the opponent's style of play during the artificial match analysis.

- *Tie outcome is neutral. There is no learning from ties* (LCA/best/win-loss-tie 2). Here, it is assumed that there is nothing to capture from a game with a tie outcome. In this situation, again there are nine updating equations whereas the previous equations change as follows.

Else if i had won and l had tied

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (22)$$

Else if i had tied and l had won

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} (b_{id}^t - b_{kd}^t)) \quad \forall d = 1, \dots, n \quad (23)$$

Else if i had tied and l had tied too,

$$x_{id}^{t+1} = b_{id}^t \quad \forall d = 1, \dots, n \quad (24)$$

Else if i had lost and l had tied



$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (b_{jd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (25)$$

Else if i had tied and l had lost

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_2 r_{1id} (b_{kd}^t - b_{id}^t)) \quad \forall d = 1, \dots, n \quad (26)$$

- *Tie outcome is randomly interpreted as win or loss (LCA/best/win-loss-tie 3).* Here, team *i* decides to interpret the tie outcome as a win or loss, in a random fashion. For example, under the case of “**Else if i had won and l had tied**” the new formation is set up as follows:

$$x_{id}^{t+1} = b_{id}^t + y_{id}^t (\psi_1 r_{1id} u_i (b_{id}^t - b_{kd}^t) + \psi_2 r_{2id} (1 - u_i) (b_{kd}^t - b_{id}^t) + \psi_1 r_{3id} (b_{id}^t - b_{jd}^t)) \quad \forall d = 1, \dots, n \quad (27)$$

where u_i is a binary valued variable. To assign a value to u_i , a random number in $[0,1]$ is generated; if it is less than 0.5, then we set $u_i = 0$, otherwise we set $u_i = 1$.

- *Tie outcome is interpreted as a win (LCA/best/win-loss-tie 4).* Such interpretation of the tie outcome models the circumstances where grabbing the tie score is vital, insomuch as tying the game may be as worth as winning the game (for example, when a team attends in a playoff in the opponent’s home). Such a view to the tie outcome is rather conservative. In this situation there are four updating equations completely similar to those were used in LCA/best. However, these equations are used under different conditions as follows:

If i had won/tied and l had won/tied, then use S/T equation (equation (9)) to set up a new team formation

Else if i had won/tied and l had lost, then use S/O equation (equation (10)) to set up a new team formation

Else if i had lost and l had won/tied, then use W/T equation (equation (11)) to set up a new team formation

Else if i had lost and l had lost too, then use W/O equation (equation (12)) to set up a new team formation

End if

- *Tie outcome is interpreted as a loss (LCA/best/win-loss-tie 5).* This case is in contrary with the previous interpretation of ties. Here we can assume a situation under which there is a tight competition between the participants and they must gather all possible scores to become the champion. In such a situation, the tie score may be worthless insomuch as even the game outcome may be treated as a pure loss. Under such a tight competition mode, the conditions for setting up a new team formation are as follows:

If i had won and l had won too, then use S/T equation (equation (9)) to set up a new team formation

Else if i had won and l had lost/tied, then use S/O equation (equation (10)) to set up a new team formation

Else if i had lost/tied and l had won, then use W/T equation (equation (11)) to set up a new team formation

Else if i had lost/tied and l had lost/tied, then use W/O equation (equation (12)) to set up a new team formation

End if

Results obtained by LCA/best under each of the above interpretations of ties are tabulated in Table 4. Figs 15 to 19 show the convergence behavior of the algorithm, on each function, under various interpretations of the tie outcome. From the results it can be inferred that both of “LCA/best/win-loss-tie 1” and “LCA/best/win-loss-tie 5” converge faster than others to the global minimum. In these algorithms, the weight given to the “approach” component is greater than the weight given to the “retreat” component which results in preservation of more intensification. Since any intensification weakens the exploration ability, we can observe that on the



Rosenbrock function, these algorithms perform a little bit weaker than the other variants, which allow more diversification. It should be noted that in “LCA/best/win-loss-tie 1” the more emphasis on the intensification is just due to a larger value chosen for ψ_2 in comparison with ψ_1 . In terms of the final solution quality, LCA/best/win-loss-tie 2 is the most effective algorithm with an acceptable performance on all functions. However, in terms of the convergence speed, this algorithm performs slowly on some functions. It is interesting to note that the updating equations used in “LCA/best/win-loss-tie 2” are a combination of the updating equations used in LCA/best, “LCA/best/Learning only from the team’s previous game” and “LCA/best/Learning only from the opponent’s previous game”. The worst result is almost reported by “LCA/best/win-loss-tie 4”. Unlike “LCA/best/win-loss-tie 1” and “LCA/best/win-loss-tie 5”, the lack of intensification in this algorithm results in a slow convergence. However, the high exploration ability of this algorithm makes it capable of finding good quality solutions for the Rosenbrock function. Unlike the tight competition mode pursued in “LCA/best/win-loss-tie 5”, which encourages more intensification, the conservative mode followed in “LCA/best/win-loss-tie 4” provides more diversification. Figs 15 to 19 demonstrate that the convergence behavior of “LCA/best/win-loss-tie 3” is similar to LCA/best. Here, there is a balance between the amount of intensification and diversification produced via inclusion of the tie outcome. Therefore, the convergence speed of “LCA/best/win-loss-tie 3” is something between the extreme cases, e.g., “LCA/best/win-loss-tie 4” and “LCA/best/win-loss-tie 5”.

Table 4. Results obtained under different interpretations of the tie outcome

Function	LCA/best/win-loss-tie 1	LCA/best/win-loss-tie 2	LCA/best/win-loss-tie 3	LCA/best/win-loss-tie 4	LCA/best/win-loss-tie 5	LCA/best (win-loss)						
f_1	$1.13E-10 \pm 3.64E-10$	18	$6.46E-11 \pm 3.46E-10$	27	$4.90E-8 \pm 2.62E-7$	14	$3.98E-8 \pm 2.17E-6$	8	$3.60E-10 \pm 7.97E-10$	16	$2.23E-9 \pm 1.15E-8$	15
f_2	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30
f_3	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30
f_4	0 ± 0	30	0 ± 0	30	0 ± 0	30	$1.63E-6 \pm 4.20E-6$	0	0 ± 0	30	0 ± 0	30
f_5	0.38 ± 0.42	0	0.08 ± 0.11	0	0.40 ± 0.92	0	0.12 ± 0.18	0	0.32 ± 0.64	0	0.06 ± 0.10	0

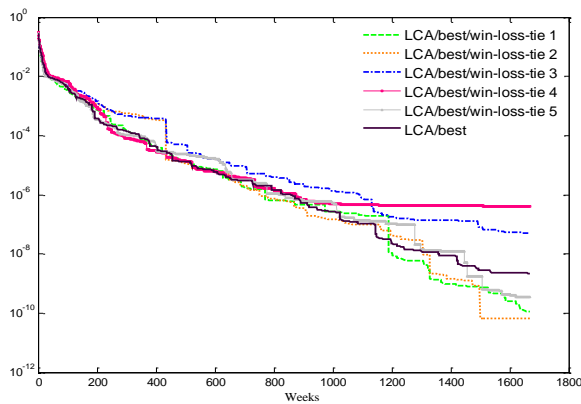


Fig 15. Progress of the mean of best function values for Shafer F6 function under different interpretations of the tie outcome.

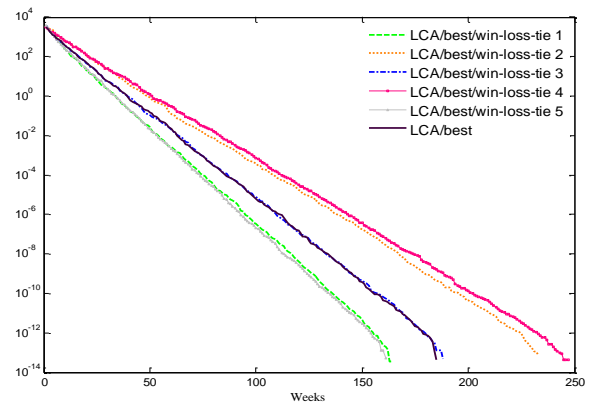


Fig 16. Progress of the mean of best function values for Sphere function under different interpretations of the tie outcome.



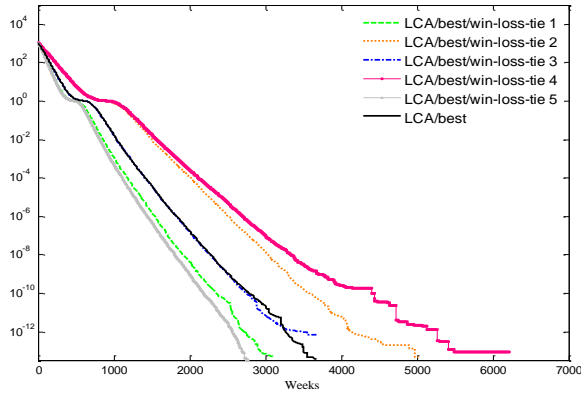


Fig 17. Progress of the mean of best function values for Griewank function under different interpretations of the tie outcome.

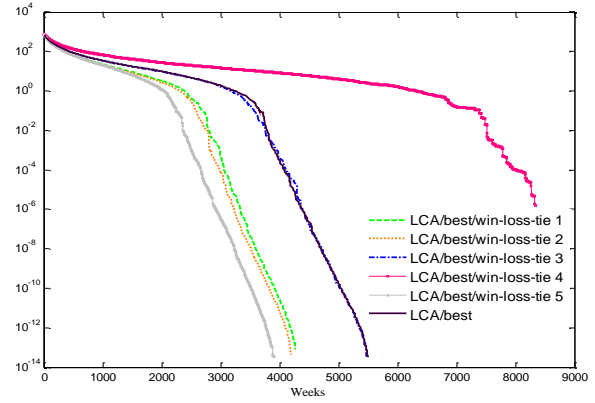


Fig 18. Progress of the mean of best function values for Rastrigin function under different interpretations of the tie outcome.

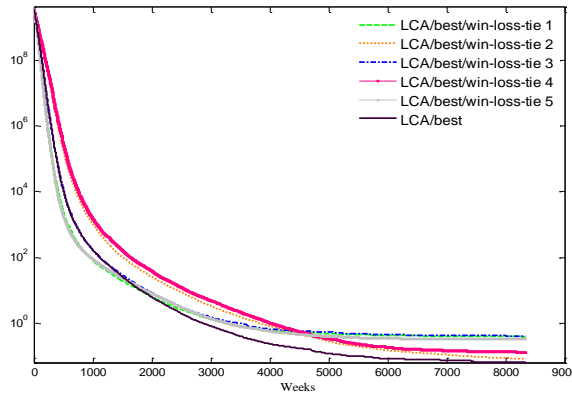


Fig 19. Progress of the mean of best function values for Rosenbrock function under different interpretations of the tie outcome.

4-3-2. Inclusion of the end season transfers

In team playing, the term “transfer” is referred to as the action taken whenever a player moves between clubs. Likewise in LCA we can introduce a transfer like operator (or a local search module) with the aim of speeding up the convergence of the algorithm. At the end of each season (i.e., every $L-1$ successive weeks, or alternately when $\text{mod}(t, L-1) = 0$) transfers are allowed for team i ($i = 1, \dots, L$), where a number of dimensions are selected randomly from B_i^t and their relevant value b_{id}^t is replaced with the corresponding value b_{md}^t taken from B_m^t (m is selected randomly from the set of indices $M_i = \{m \mid m \neq i, f(B_m^t) < f(B_i^t)\}$). To decide whether the d^{th} element of B_i^t should change or not, a random number is generated within $[0, 1]$; if it is less than or equal to a threshold parameter T_r , then an index m is selected randomly from M_i and the value of b_{id}^t is updated by the value of b_{md}^t . The transfer operator only allows that the components of better solutions being inserted into B_i^t (this condition is enforced by $f(B_m^t) < f(B_i^t)$). The procedure of the transfer operator is as follows:

For $i = 1 : L$

$$M_i = \{m \mid m \neq i, f(B_m^t) < f(B_i^t)\};$$



```

For  $d=1:n$ 
  If  $rand < T_r$ 
    Randomly select an index  $m$  from  $M_i$ ;
     $b'_{id} \leftarrow b'_{md}$ ;
  End if
End for
End for

```

Table 5 summarizes the results obtained by LCA/best under different levels of T_r . As the results indicate, the transfer operator can have a significant role in reducing the number of function evaluations before reaching the optimum. Through insertion of the elements of better solutions in a weaker solution, the transfer operator steers the search quickly toward promising areas. However, this operator may not be effective on all functions. As our earlier experiments indicated, the Rosenbrock function is the one in which the quality of the final solution is improved if a proper level of diversification is preserved. Therefore, it is not surprising to see that using the transfer operator in LCA/best significantly deteriorates the quality of the final solutions found for this function. Figs 20 to 24 show the progress of the mean of best function values under different values of T_r . From these plots one can observe that an increment in the level of T_r results in the unsuccessfulness of the algorithm in reaching always the global minimum, for at least one additional function. Therefore, the small values of T_r , e.g., 0.1, are more preferred. Large values of T_r may cause the population being rapidly homogenous and the algorithm finishes with a premature convergence.

Table 5. Results obtained under different levels of T_r

Function	T_r											
	0.1	0.3		0.5		0.7		0.9		LCA/best ($T_r = 0$)		
f_1	0 ± 0	30	0 ± 0	30	0 ± 0	30	$3.23E-4 \pm 1.77E-3$	29	$9.71E-4 \pm 2.96E-3$	27	$2.23E-9 \pm 1.15E-8$	15
f_2	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30	0 ± 0	30
f_3	0 ± 0	30	0 ± 0	30	$2.46E-4 \pm 1.35E-3$	29	$2.58E-3 \pm 1.00E-2$	21	0.14 ± 0.21	0	0 ± 0	30
f_4	0 ± 0	30	0.66 ± 0.79	15	2.58 ± 1.09	1	6.72 ± 2.58	0	12.51 ± 3.14	0	0 ± 0	30
f_5	9.16 ± 13.71	0	39.89 ± 34.74	0	85.72 ± 56.13	0	223.43 ± 142.68	0	5024.06 ± 7604.34	0	0.06 ± 0.10	0

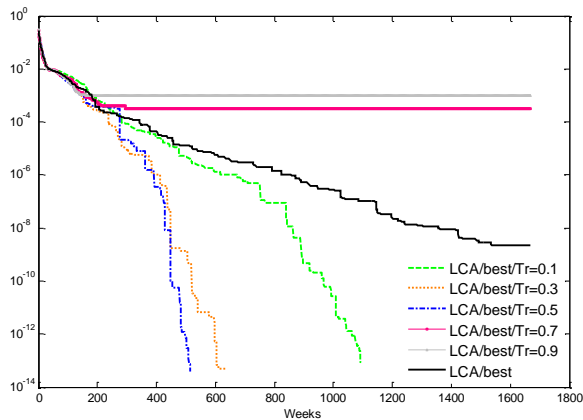


Fig 20. Progress of the mean of best function values for Shafer F6 function under different levels of T_r .

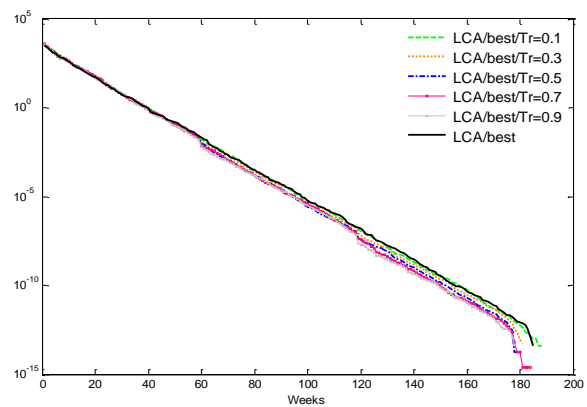


Fig 21. Progress of the mean of best function values for Sphere function under different levels of T_r .



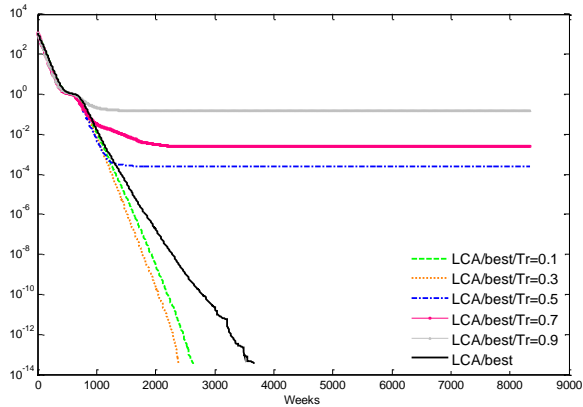


Fig 22. Progress of the mean of best function values for Griewank function under different levels of T_r .

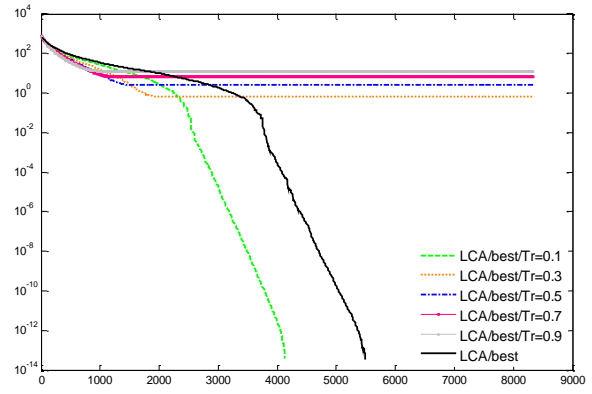


Fig 23. Progress of the mean of best function values for Rastrigin function under different levels of T_r .

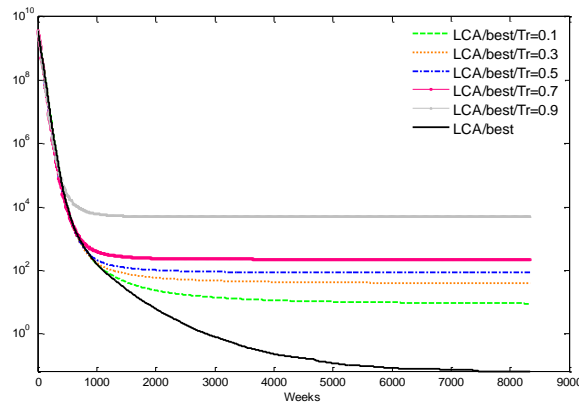


Fig 24. Progress of the mean of best function values for Rosenbrock function under different levels of T_r .

5. Conclusion and future works

The league championship algorithm (LCA) has been introduced as a sport driven metaheuristic for numerical function optimization. LCA is a population based stochastic search methodology that mimics the sporting competitions in a sport league. At the heart of LCA is the artificial match analysis process where the new solutions are generated using a metaphorical SWOT analysis, which is typically followed by coaches during the planning for the next game. Some analysis was carried out to verify the rationale of the algorithm and the suitability of the updating equations, empirically. Some add-on modules were also introduced into LCA with the aim of possibly enhancing the convergence speed of the algorithm toward the global minimum.

LCA is in its infancy, and much remains to be done to realize its full potency. It is therefore the author's hope that this paper inspires future works on developing LCA's theory and applications. Choosing a value for the control parameters of any optimization algorithm is often subjective, reflecting the user's experience and insight with a particular problem. To eliminate the need of the case based tuning of the control parameters, self-adaptive strategies could be employed in LCA as they are used in other evolutionary algorithms. Finally, the performance of LCA can be further tested on the real world engineering optimization problems addressed in the literature. Besides, a comparative study would be worth to conduct to measure the potency of LCA in comparison with other state of the art global optimization algorithms.



The appendix: Simulation of the number of changes imposed to the best formations

A geometric random variable X is distributed by the probability and cumulative density functions as follows:

$$X \sim \text{Geometric}(p_c): P(X = x) = p_c(1 - p_c)^{x - q_0} \quad x = q_0, q_0 + 1, \dots$$

$$F_X(x) = 1 - (1 - p_c)^{x - q_0 + 1} \quad x = q_0, q_0 + 1, \dots$$

where p_c is the distribution parameter. When X is bounded to n , the probability and cumulative density functions are changed as follows:

$$X \sim \text{TruncatedGeometric}(p_c): P(X = x) = \frac{p_c(1 - p_c)^{x - q_0}}{1 - (1 - p_c)^{n - q_0 + 1}} \quad x = q_0, q_0 + 1, \dots, n$$

$$F_X(x) = \frac{1 - (1 - p_c)^{x - q_0 + 1}}{1 - (1 - p_c)^{n - q_0 + 1}} \quad x = q_0, q_0 + 1, \dots, n$$

To develop a formula for generating a random number from the truncated geometric distribution, we have [17]:

$$F_X(x-1) < r \leq F_X(x) \Rightarrow \frac{1 - (1 - p_c)^{x - q_0}}{1 - (1 - p_c)^{n - q_0 + 1}} < r \leq \frac{1 - (1 - p_c)^{x - q_0 + 1}}{1 - (1 - p_c)^{n - q_0 + 1}}$$

$$\Rightarrow \frac{\ln(1 - (1 - (1 - p_c)^{n - q_0 + 1})r)}{\ln(1 - p_c)} + q_0 - 1 \leq x < \frac{\ln(1 - (1 - (1 - p_c)^{n - q_0 + 1})r)}{\ln(1 - p_c)} + q_0$$

$$\Rightarrow X = \left\lceil \frac{\ln(1 - (1 - (1 - p_c)^{n - q_0 + 1})r)}{\ln(1 - p_c)} \right\rceil + q_0 - 1 \quad (28)$$

where, r is a uniformly distributed random variable within $[0,1]$. Fig 25 shows the histogram of the number of changes made in the best formation for various values of p_c . As can be seen, as the value of p_c becomes greater, the maximum number of changes becomes smaller such that for $p_c = 0.99999$ the number of changes is always one. Each histogram in Fig 25 has been plotted based on 1000 random numbers generated by equation (30) with $q_0 = 1$.

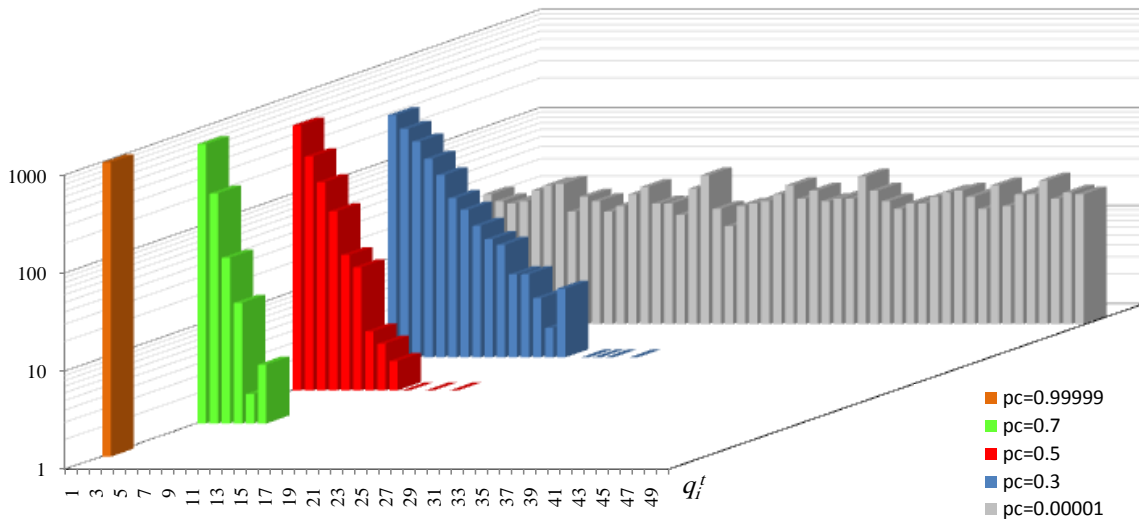


Fig 25. Effect of various values of p_c on the number of changes imposed to the best formation.



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